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Limiting temperature of hadrons using states predicted from κ-deformed Poincaré algebra

Jishnu Dey[2,3], Siddhartha Bhowmik[1], Kanad Ray[1,2,3] and Subharthi Ray[1,2,3]

[1]Department of Physics, Presidency College, Calcutta-700 073, India
[2]Azad Physics Centre, Department of Physics, Maulana Azad College, Calcutta-700 013, India

Abstract . The experimental hadronic density of states dN/dm, assumed to be a sum of normalized Breit-Wigner distributions and plotted as a function of the hadron mass *m*, fails to show a Hagedorn like growth beyond 2 GeV, probably due to a lack of data Experimental hadronic states are fitted using κ-deformed Poincaré algebra and the fit is used to extrapolate for including states not detected For the theoretical density of states the plot is a straight line in the log scale even beyond 2 *GeV* with a limiting temperature of 400 *MeV*.

Keywords . Hadrons, Poincaré algebra, limiting temperature

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Deformed Poincaré algebra, (dPa in short), keeps the three dimensional rotation and the translation subgroups undeformed while the algebra of Lorentz boosts is modified, both for bosons and fermions. The relevant q-deformation parameter is called k in this case and when this goes to infinity we recover the undeformed algebra. The k-deformed Dirac equation has recently been found [1]. Extensive applications of dPa have been carried out to see what would be its impact on the standard theories governed by the ordinary quantum special relativity. The following problems have been studied.

a) The definition of mass with different non-relativistic limits [2],

b) the non-additivity of masses and its relation to the interesting dark matter puzzle [3],

c) the classical electrodynamics problem of finding the acceleration of charged particle in a one-dimensional homogenous electric field [4],

d) gauging the deformed Dirac equation, applying it to the quantum relativistic hydrogen atom and solving the Dirac-Coulomb problem [5],

e) calculating the Landau deformed levels [6],

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f) explanation of the flattening of the experimental hadron spectrum [7], [8],

g) application of the new mass-energy relations of k-deformed algebra to the model of Nambu and Jona-Lasinio, *now with a natural cut-off $1/\varepsilon$ provided by the theory* [9].

h) Quite recently it has been suggested [10] that matter and radiation can be created in the confined vacuum of a quantum field whose spacetime symmetries are governed by Poincaré algebra. It is claimed that the creation rate goes to zero when the deformation disappears. We shall have occasion to come back to a further discussion of this very interesting paper.

i) The flattening of hadron spectrum, explained by the deformed algebra in the case (f), seems to lead to interesting smooth phase transitions at finite T [11].

From one of these studies, namely the case (d), it turns out that for negligible deformation, the normal Dirac equation is recovered. Expansion in the deformation parameter gives the result that the first order effect vanishes identically [5]. This means clearly that there is no change in the energy spectrum in the first order of perturbation theory. This does not happen for the deformed Landau levels [6], which are expected to shift already in first order perturbation theory. As we can see, people are getting interested to see how a determined theory or equation behaves under a new symmetry structure generated by a group deformation.

In the present paper, we use the formalism described above to fit and extrapolate the observed baryons. Some of the mesons were already fitted [8], we fit the rest, *viz*. the K and the K^* and the η and the η'. We use

$$M(n, L, S, L) = \frac{2}{\varepsilon}\sinh^{-1}\left[\left(\frac{\varepsilon}{2}\right)^2\left(\frac{L}{\alpha'}+\frac{n}{\beta'}+\frac{S}{\gamma'}+\frac{J}{\delta'}\right)+\sinh^2\left(\frac{m\varepsilon}{2}\right)\right]^{1/2}, \quad (1)$$

where L, S and J stand for orbital, spin and total angular momentum and n is the quantum number for radial excitation.

The value of ε is fixed once for all at 0.915 GeV^{-2}. For the π, ρ and ω we use $m = 0.138$ GeV. This implies that the $\rho - \omega$ are spin-excitations of the pion. In the same way we use $m =$ 0.494 GeV for K and K^*. The spin parameters $\gamma' = 2.35$ GeV^{-2} and $\delta' = 5.5$ GeV^{-2} are also unaltered. The parameters $\alpha' = 0.7$ GeV^{-2} and $\beta' = 0.5292$ GeV^{-2} for π, ρ and ω are changed to 0.679 and 0.44 for the strange mesons. For the η and η', m is 0.547 and 0.958 GeV and $\alpha' = 0.99$ GeV^{-2} and $\beta' = 0.67$ GeV^{-2}.

For baryons the δ', α', β', r' and m are given in Table 2.

A series of papers [12-14], deal with the densities of observed mesons and baryon states and their possible relationship with hadron-scale string theories. The frustration involved in this kind of work stems from the fact that the experimental states are known only upto ≈ 2.5 GeV and even in this region probably many states are not experimentally identified. Thus the total density of hadrons in Figure 1 plotted in log scale fails to grow linearly beyond 2 GeV, and this is 'likely to be a reflection of current experimental limitations' [13]. Since we are able to predict meson and baryon states, we check this result. Indeed Figure 1 with the extrapolated states goes like a straight line with a slope of $T_H = 400$ MeV, slightly larger than the values of 250 [13] or 300 [14] MeV, but quite in line with the expectation of Cudell and Dienes. Note that the value $T_H \sim 160\,MeV$ for the Hagedorn temperature is too low to agree with the central charge of the effective QCD string [14].

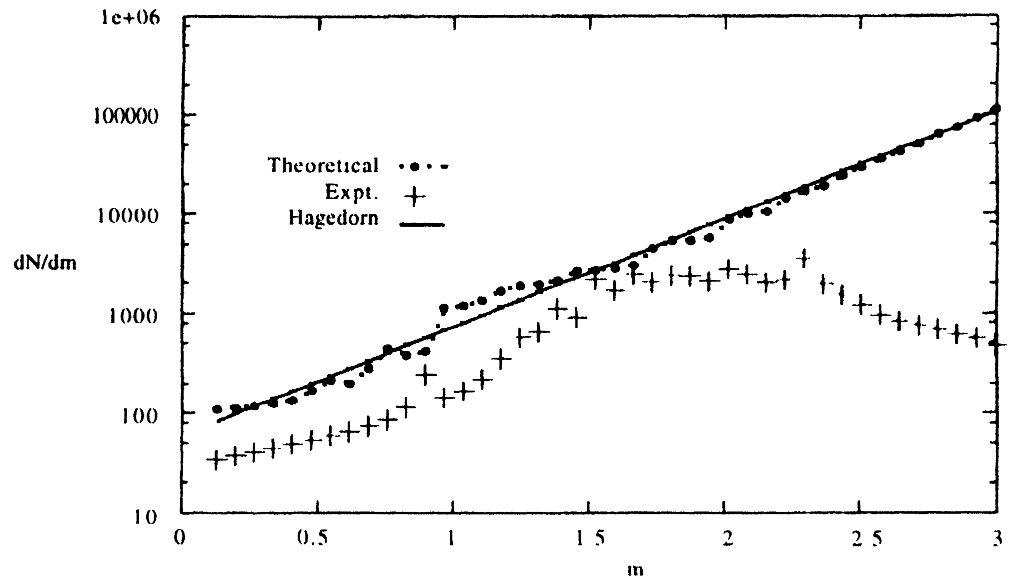

Figure 1. Comparison of experimental ('plus'-s, ++) and theoretically predicted density of states (dots) The straight line gives ~ $exp\,(m/T_H)$

To Plot Figure 1 we use :

$$\frac{dN}{dm} = \frac{1}{2\pi}\sum_i W_i \frac{\Gamma_i}{(m - M_i)^2 + \frac{\Gamma_i^2}{4}}, \tag{2}$$

where the masses m and widths Γ are taken from [15] for the experimental curve (with ++). For the theoretical curve (with dots). We use the masses from eq. (1) with widths 8.5 MeV below 1 GeV and 55 MeV above. This choice makes the dots relatively smooth. The important point is that from 2 to 3 GeV, the theoretical curve smoothly fits onto ~ exp (m/T_H) with $T_H = 400\,MeV$.

Eq. (1) fits the experimental states rather well. The π, η, η' and K are fitted and therefore left out of Table 1. Note the good fit to the ρ, ω, K^*, h (first and second), radial excitations of π at 1.3, K at 1.46, (ρ, ω) at (1.7, 1.6), – even ρ_5 at 2.35 and K^*_4 at 2.045 (all in GeV).

For the baryons the ground states, which are fitted, are also put in the Table 3 to enable the reader to identify the sets easily. For nucleon states the Roper at 1.44 and its higher radial excitations are well fitted, but there is the well-known problem of fitting the second S11 state, while the third S11 is well fitted. The other angular excitations are also reasonably well fitted and we are anticipating new experimental data to come from CEBAF (Jefferson centre). For the strange baryons the fit is similar in quality.

We next turn to thermodynamics of the hadron gas. In [11], it was suggested that there is a smooth phase transition in energy density in the extrapolated hadron spectrum using deformed Poincaré algebra. However, it is now clear to us that thermodynamic quantities are ill-defined and the sum over particle states in them do not converge beyond T_H. It is also clear

Table 1. Meson masses from our model compared to experiment.

Meson state	Ours	Expt	Meson state	Ours	Expt
ρ, ω	775	77, .782	ρ, ω	1 692	1 7, 1.6
π2	1.641	1 67	b1	1.213	1.235
ρ3, ω3	1 764	1 69, 1 67	a4, f4	2.031	2.04, 2.05
ρ5	2.243	2 35	a1, f1	1.347	1.26, 1.285
a2, f2	1.398	1 32, 1.27	ρ3	2.071	2.25
f2	1.815	1.81	ρ, ω	1.472	1.45, 1.42
ρ	1 865	2.15	π	1.303	1.3
π	1.754	1 8	π2	1 982	2 1
f2	2 11	2.15	f0	2.059	2.2
f2	2 339	2.3	f4	2.276	2.3
a6, f6	2 42	2.45, 2.51	η2	1 939	1.87
h1	1 167	1.17	η	1.461	1.44
η	1 269	1.295	h1	1.38	1.38
η	1 652	1.76	K1	1.3012	1 27
K*	.899	.892	K*	1 755	1.68
K1*	1.423	1 4	K3*	1 822	1.78
K2	1.707	1.77	K5*	2.291	2.38
K4*	2 083	2 045	K2*	1.927	1 98
K2*	1.471	1.43	K	2 023	2.1
K*	1.617	1.68	K	1.929	1.83
K	1.474	1 46	–	-	–

that dPa applies only to the internal structure of the hadrons. However, below T_{II} we can still calculate the free energy F (and the energy E), of the hadron gas :

$$F(T) = \frac{T}{2\pi^2}\int_0^{\infty} k^2 \sum_{nLSJ} g_{nLSJ}\, ln\left[1 - e^{-E/T}\right] dk \tag{3}$$

Table 2. Different value of the parameters for the baryons.

Baryon Name	δ'	α'	β'	γ'	m
Nucleon	−5.5	58	685	2.6	.889
Delta	−7.2	.48	.685	2.6	1.12
Lambda	−5.5	.7	.8	2.6	1.077
Sigma	−5.5	.7	.8	2.6	1.153
Cascade	1.15	.7	.8	2.6	1.102

for Bose gas and

$$F(T) = -\frac{T}{2\pi^2}\int_0^\infty k^2 \sum_{nLSJ} g_{nLSJ}\, ln\left[1+e^{-E_k/T}\right] dk \tag{4}$$

for Fermi gas with

$$E_k = \sqrt{k^2 + M(n, L, S, J)^2}\ . \tag{5}$$

Hence, its entropy $S \equiv (E-F)/T$ is known. We plot $(E-F)/E$ in Figure 2. This quantity obviously starts from zero. It fast approaches the value 1.2 for mesons, almost independent of the temperature, tantalizingly close to the ratio 4/3 as in the case of massless quarks and gluons. There is a little dip in the curve which (Figure 2) we do not understand at present and do not wish to comment on. For baryons the ratio is somewhat less, close to 1.1, but almost independent of T in the range displayed. Bearing in mind that at least the strange quark *is* massive, it may be possible that somewhere below the T_H a realistic quark-gluon description sets in rather smoothly. We find it interesting that the hadron gas, with the high occupation probability of massive resonances, still has a $(E-F)/E$ ratio which is close to that of nearly massless particles.

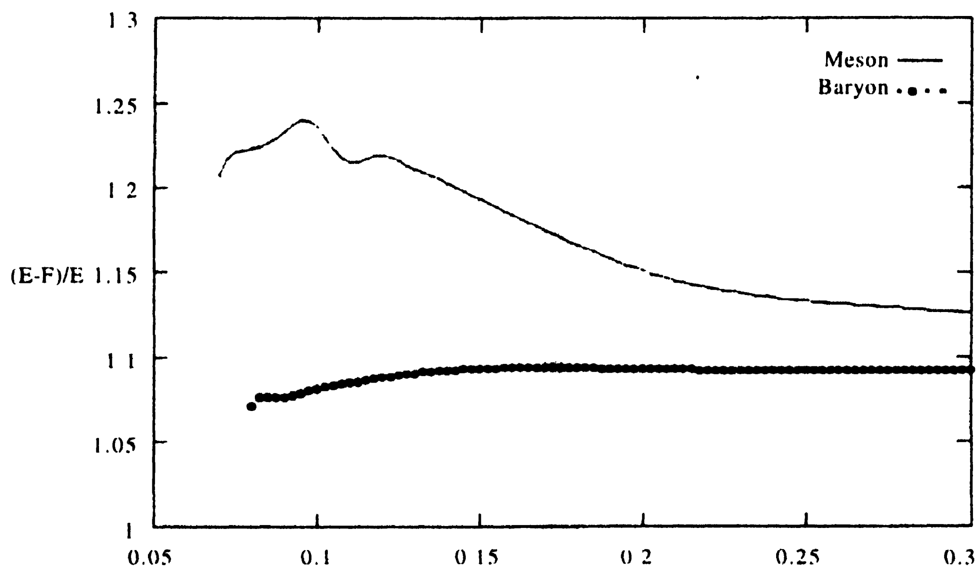

Figure 2. The calculated values of $\frac{E-F}{E}$ for mesons and baryons.

In summary, we find the string theorist's expectation that the Hagedorn T_H is almost double the conventional value ~ 160 *MeV* is borne out for hadronic states generated by *k*-deformed Poincaré algebra. This work was supported in part by a grant from the Department of Science and Technology, Govt. of India, two of the authors (Rays) hold appointment under this grant.

Table 3. Baryon masses from our model compared to experiment.

Nucleon state	Ours	Expt	Nucleon state	Ours	Expt	Nucleon state	Ours	Expt
P11	0.939	0.939	P11	1.765	1.71	P11	2.01	2.1
P11	1.441	1.44	P13	1.829	1.72	G17	2.046	2.19
D13	1.463	1.52	P13	2.061	1.9	D15	2.26	2.22
S11	1.508	1.535	F17	2.269	1.99	H19	2.248	2.22
S11	1.814	1.65	F15	2.034	2.0	G19	2.435	2.25
F15	1.796	1.68	D13	2.022	2.08	I1,11	2.417	2.6
D13	1.781	1.7	S11	2.049	2.09	K1,13	2.563	2.7
Delta state	**Ours**	**Expt**	**Delta state**	**Ours**	**Expt**	**Delta state**	**Ours**	**Expt**
P33	1.232	1.232	P33	2.087	1.92	H39	2.51	2.3
P33	1.621	1.6	D35	1.976	1.93	D35	2.327	2.35
S31	1.775	1.62	D33	1.997	1.94	F37	2.522	2.39
D33	1.748	1.7	F37	2.048	1.95	G39	2.296	2.4
S31	2.018	1.9	F35	2.256	2.0	H3,11	2.497	2.42
F35	2.068	1.905	S31	2.214	2.15	I3,13	2.666	2.75
P31	2.106	1.91	G37	2.311	2.2	K3, 15	2.812	2.95
Lambda state	**Ours**	**Expt**	**Lambda state**	**Ours**	**Expt**	**Lambda state**	**Ours**	**Expt**
P01	1.116	1.116	S01	2.006	1.8	F05	1.978	2.09
S01	1.539	1.407	P01	1.766	1.81	G07	1.977	2.1
D03	1.495	1.52	F05	1.765	1.815	D03	1.978	2.325
P01	1.496	1.6	D05	2.005	1.83	H09	2.153	2.35
S01	1.799	1.67	P03	1.799	1.85	-	-	-
D03	1.765	1.685	F07	2.177	2.02	-	-	-
Sigma state	**Ours**	**Expt**	**Sigma state**	**Ours**	**Expt**	**Sigma state**	**Ours**	**Expt**
P11	1.189	1.189	D15	2.034	1.77	S11	2.035	2.0
D13	1.544	1.58	P11	1.803	1.77	F17	2.202	2.025
S11	1.586	1.62	P13	1.835	1.84	F15	2.008	2.07
P11	1.544	1.63	P11	2.009	1.88	P13	2.035	2.08
D13	1.803	1.665	D13	2.009	1.9	G17	2.008	2.1
S11	1.835	1.73	F15	1.802	1.9	-	-	-
Cascade state	**Ours**	**Expt**	**Cascade state**	**Ours**	**Expt**	**Cascade state**	**Ours**	**Expt**
P11	1.315	1.315	D13	1.837	1.823	-	-	-

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